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The isospin dependence of the nucleus - nucleus inelastic cross section at high energy

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t The isospin dependence of the nucleus - nucleus inelastic cross section at high energy is investigated within the multiple scatteri[illegible] he multiple integrals are evaluated by Monte Carlo method as well as by the optical limit approximation of the Glauber model Calculatio [illegible]med for $^{14-[illegible]}$N, $^{16-24}$O and $^{18-26}$F isotopes colliding with carbon target around 1 GeV It is found that rms radii and the density distributions sh[illegible] ucture of $^{[illegible]}$N $^{[illegible]}$O and ^{24}F

words Nucleus-nucleus scattering, inelastic cross section, isospin dependence, Monte-carle method

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Introduction

Glauber model has been applied by many authors to study ion collisions and for describing a number of interaction sses over a wide range of energies The model gives the us-nucleus interaction in terms of interaction between the stituent nucleons with a given density distribution The lel is a semiclassical model picturing the nuclear collision in impact parameter representation which works well at high gy Now, hadron-nucleus and nucleus-nucleus interactions widely studied at high energy experimental physics and the ve been analysed using Glauber as well as other different histicated models In this work, we applied two oximations of the Glauber model in order to deduce the rms s well as the matter-density distribution of neutron-rich opes using intermediate and high energy nucleus-nucleus ions, where the many-body multiple integral of the Glauber roximation is calculated exactly by Monte Carlo method as l as the optical limit approximation

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2. Theoretical description

2.1 *Monte Carlo approach for Glauber model*

The scattering amplitude of two nuclei with the mass number A and B in the impact parameter representation is given by [1-12]

$$F(\boldsymbol{b})\Big|_{A(i\to f)B(i\to f)} = \Big\langle \Psi_A^f, \Psi_B^f$$

$$1-\prod_{j=1}^{A}\prod_{k=1}^{B}\left[1-\gamma(\boldsymbol{b}-s_j+\tau_k)\right]\Big|\Psi_B^i, \Psi_A^i\Big\rangle, \qquad (1)$$

where $\boldsymbol{b}$ is the impact parameter vector The angle brackets (< >) mean the average over the initial Ψ_A^i, Ψ_B^i and the final Ψ_B^f, Ψ_A^f state wave functions of nuclei A and B $\gamma(\boldsymbol{b})$ is amplitude of elastic nucleon-nucleon (NN) scattering in the impact parameter representation such that

$$\gamma(\boldsymbol{b}) = \frac{1}{2\pi i p}\int e^{i\boldsymbol{q}\cdot\boldsymbol{b}} f(q)d^2q, \qquad (2)$$

where p is the momentum of nucleus A per nucleon in a system and the target nucleus B is at rest, $f(q)$ is the NN-elastic scattering amplitude in the momentum representation $\{s_A\}$ and $\{\tau_B\}$ are the coordinates of the nucleons with regard to the centers of mass of nuclei A and B, respectively, in the plane of the impact parameter (plane perpendicular to the momentum $\boldsymbol{p}$) Using (1), one can find different nucleus-nucleus interaction characteristics. For example, on the simple assumption that

$$|\Psi'_A|^2 = \prod_{j=1}^{A} \rho_A(s_j, z_j), |\Psi'_B|^2 = \prod_{j=1}^{B} \rho_B(\tau_j, \xi_j), \quad (3)$$

where ρ_A and ρ_B are the one-particle density of nuclei A and B, the inelastic cross section is σ^{in}_{AB} given as

$$\sigma^{in}_{AB} = \int d^2b \left\{ 1 - \prod_{i=1}^{A}\prod_{j=1}^{B}\left[1 - \gamma(\boldsymbol{b} - s_i + \boldsymbol{\tau}_j) - \gamma^*(\boldsymbol{b} - s_i + \boldsymbol{\tau}_j)\right.\right.$$

$$\left.\left. +\gamma(\boldsymbol{b} - s_i + \boldsymbol{\tau}_j)\gamma^*(\boldsymbol{b} - s_i + \boldsymbol{\tau}_j)\right]\right\}$$

$$\left\{\prod^{A} \rho_A(s_i, z_i) d^3r_i\right\}\left\{\prod^{B} \rho_B(\tau_i, \xi_i) d^3t_i\right\} \quad (4)$$

The main problem of the calculation of this expression is the calculation of the profile function

$$\Gamma(b) = \int \left\{1 - \prod_{i=1}\prod_{j=1} (1 - p_{ij})\right\} \times \left\{\prod \rho_A(r_i)\, d^3r_i\right\}$$

$$\times\left\{\prod_{i=1}^{B} \rho_B(t_i)\, d^3t_i\right\}, \quad (5)$$

where $p_{ij} = \gamma(\boldsymbol{b} - s_i + \boldsymbol{\tau}_j) + \gamma^*(\boldsymbol{b} - s_i + \boldsymbol{\tau}_j) - \gamma(\boldsymbol{b} - s_i + \boldsymbol{\tau}_j) \times \gamma^*(\boldsymbol{b} - s_i + \boldsymbol{\tau}_j)$ For an interpretation of the experimental data within the framework of the Glauber approach, one needs an effective procedure for evaluating the expressions like (5) It was suggested in [9] that the Monte Carlo method should be used for this purpose We consider the integrand containing nuclear densities as a probabilistic measure, then one can easily construct the Monte Carlo estimation of the multiple integral in the form

$$G(\boldsymbol{b}\{r_A\},\{r_B\}) = 1 - \prod\prod (1 - p_{ij}) \quad (6)$$

The mean value of $G(\boldsymbol{b}\{r_A\},\{r_B\})$ corresponds to the profile function $\Gamma(\boldsymbol{b})$,

$$\Gamma(\boldsymbol{b}) = \frac{1}{N}\sum_{i=1} G(\boldsymbol{b},\{r_A\},\{r_B\}), \quad (7)$$

where N is the number of sets of nucleon coordinates of nuclei A and B, sampled from the distribution defined by probabilistic measure

$$d\mu[\rho_A, \rho_B] = \left[\prod^{A} \rho_A(r_i) d^3r_i\right]\left[\prod^{B} \rho_B(t_j) d^3t_j\right]$$

Practically, N of the order of one hundred is sufficient to obtain the value of an inelastic cross section with the accuracy of several percent

2 2 The optical limit approximation of the Glauber model

The total inelastic cross section can be written as

$$\sigma_{in}^{AB} = 2\pi \int bdb(1 - s^{AB}),$$

and the scattering matrix can be written as

$$|S(b)|^2 = s^{AB} = (1 - T(b)\,\sigma_{NN})^{AB},$$

where σ_{NN} is the total NN cross section In the optical approximation, we can write

$$|S(b)|^2 = \exp(-T(b)\,\sigma_{NN} AB)$$

In the coordinate space, the t-matrix can be written as

$$T(b) = \int \rho_A^z(\boldsymbol{b}_A) d\boldsymbol{b}_A \rho_B^z(\boldsymbol{b}_B) d\boldsymbol{b}_B t(\boldsymbol{b} - \boldsymbol{b}_A + \boldsymbol{b}_B)$$

It is a four-dimensional integration two over the impact parameter b_A and two over b_B. It is convenient to write it in momentum space as

$$T(b) = \frac{1}{(2\pi)^2}\int \rho_A^z(\boldsymbol{b}_A) d\boldsymbol{b}_A \rho_B^z(\boldsymbol{b}_B) d\boldsymbol{b}_B$$

$$\times \exp(-i\boldsymbol{q}\cdot(\boldsymbol{b} - \boldsymbol{b}_A + \boldsymbol{b}_B))\, f_{NN} d^2q$$

Here, f_{NN} is the q-dependence of NN scattering amplitude by

$$t(b) = \frac{1}{4\pi^2}\int \exp(-i\boldsymbol{q}\cdot\boldsymbol{b})\, f_{NN}(q) d^2q,$$

$$T(b) = \frac{1}{4\pi^2}\int \exp(-i\boldsymbol{q}\cdot\boldsymbol{b}_A)\rho_A^z(\boldsymbol{b}_A)\exp(i\boldsymbol{q}\cdot\boldsymbol{b}_A) d\boldsymbol{b}_A \rho_B^z(\boldsymbol{b}_B)$$

$$\times \exp(-i\boldsymbol{q}\cdot\boldsymbol{b}_B) d\boldsymbol{b}_B f_{NN}(q) d^2q$$

$$= \frac{1}{4\pi^2}\int \exp(-i\boldsymbol{q}.\boldsymbol{b})\, S_A(\boldsymbol{q})\, S_B(-\boldsymbol{q}) f_{NN}(q) d^2q$$

$$= \frac{1}{2\pi}\int J_0(qb) S_A(\boldsymbol{q}) S_B(-\boldsymbol{q}) f_{NN}(q) q dq,$$

$S_A(q)$ and $S_B(-q)$ are the Fourier transforms of the densities The function $f_{NN}(q)$ is the Fourier transform profile function $t(b)$ The profile function for the NN ing can be taken as delta if the nucleons are point particles eral it is taken as a Gaussian function of width r_0 as

$$t(b) = \exp\left(-b^2/r_0^2\right)/\pi r_0^2 \tag{16}$$

$$\int \exp(iq\, b)t(b)db = \frac{1}{\pi r_0^2}\int \exp(iq\, b)\exp\left(-b^2/r_0^2\right)db$$

$$= \exp\left(-r_0^2 q^2/4\right), \tag{17}$$

r_0 is the range parameter and σ_{NN} is the nucleon-n inelastic cross section

lculation of $T(b)$ using the Gaussian densities

lear densities are assumed to be of the Gaussian shape

$$\rho_i(r) = \rho_i(0)\exp\left(-r^2/a_i^2\right), \quad (i = 1,2), \tag{18}$$

the parameters $\rho_i(0)$ and a_i are adjusted to reproduce rimentally determined surface texture of the nucleus integrated density will be

$$\rho_i(b) = \rho_i(0)\sqrt{\pi}\, a_i \exp\left(-b^2/a_i^2\right) \tag{19}$$

n the momentum representation, we can write it as

$$S_i(q) = \rho_i(0)\left(\sqrt{\pi}a_i\right)^3 \exp\left(-q^2 a_i^2/a_i^2\right) \tag{20}$$

overlap integral $T(b)$ can be written as

$$T(b) = \frac{1}{4\pi^2}\int \exp(-iq\, b)\, S_1(q)\, S_2(-q) f_{NN}(q)\, d^2q$$

$$\pi^3 \rho_1(0)\rho_2(0)a_1^3 a_2^3 \int \exp(-iq\, b)\exp\left(-q^2a^2/4\right)d^2q\,, \tag{21}$$

$a^2 = a_1^2 + a_2^2 + r_0^2$ Performing q integration, we get

$$T(b) = \pi^2 \frac{\rho_1(0)\,\rho_2(0)a_1^3a_2^3}{a_1^2 + a_2^2 + r_0^2}\exp\frac{-b^2}{a_1^2 + a_2^2 + r_0^2} \tag{22}$$

esults and discussion

ıs radii of the sequences of N and O isotopes are shown res 1 and 2, respectively, and the density distribution of opes is shown in Figure 3 These figures are calculated by many-body Monte Carlo method as well as optical limit of the Glauber approximation, as described in Section 2, and using Table 1 for the reaction cross section As shown in these figures, the rms radii, in general, increase with increasing the neutron number The sudden increase at ^{22}N and ^{23}O of the rms radii as well as in the density distribution, indicates a halo structure for these neutron-rich nuclei The results of the Monte Carlo approach is better than those of the optical limit approximation This could be due to the Gaussian shape used in the optical

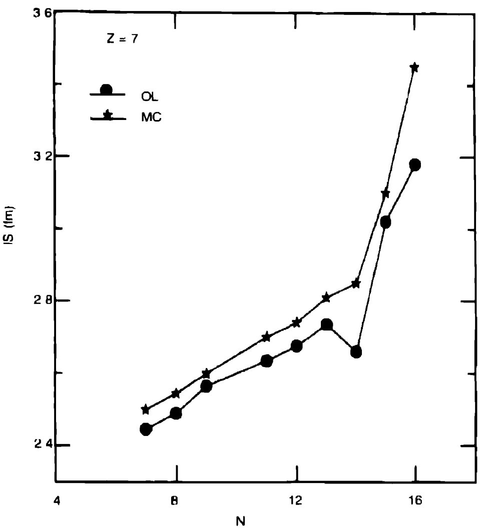

Figure 1 RMS radii of the sequences of N isotopes

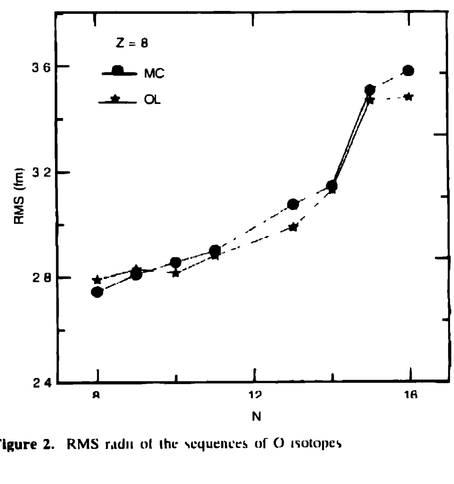

Figure 2. RMS radii of the sequences of O isotopes

limit which could not handle the isospin dependence of the reaction cross section, i e could not describe halo's or skip of exotic nuclei Moreover, the parameters of the nucleon-nucleon amplitude need to be readjusted in order to get good description of neutron-rich nuclei

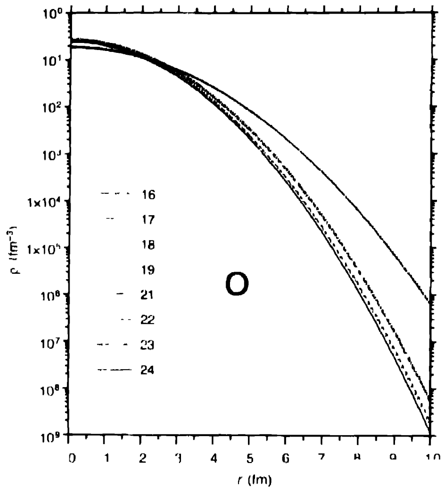

Figure 3 Density distribution of O isotopes

In this calculation, we deduced the radii of the nuclei by two Glauber-model approximations, the optical model and many-body Monte Carlo method for calculating the Glauber integral We also deduced the effective nucleon densities which describe the reaction cross section σ_R in a better way The deduced matter radii for ^{22}N and ^{23}O are larger than those of their neighbors and the resulting nucleon densities show a long tail or a halo structure of nuclei as indicated in the experiments

Table 1. Reaction cross section σ_R of the systems $^{14-23}$N, $^{16-24}$O, $^{18-26}$F + ^{12}C and RMS matter radii calculated in the multiple scattering theory Monte Carlo method and the optical limit of the Glauber model. The experimental data are taken from [8]

Nucleus	Energy	σ_R^{Exp} (mb)	R^m_{rms} (OL)(fm)	R^m_{rms}
^{14}N	965	932±9	2.768	
^{15}N	965	930±30	2.682	
^{16}N	1020	969±32	2.7312	
^{18}N	1020	1046±8	2.878	
^{19}N	1005	1076±9	2.927	
^{20}N	950	1121±17	3.053	
^{21}N	1005	1114±9	2.952	
^{22}N	965	1245±49	3.329	
^{23}N	920	1399±98	3.735	
^{16}O	970	982±6	2.80	
^{17}O	970	1010±15	2.839	
^{18}O	1050	1032±26	2.817	
^{19}O	970	1066±9	2.912	
^{21}O	980	1098±11	2.915	
^{22}O	965	1172±22	3.105	
^{23}O	960	1308±16	3.462	
^{24}O	965	1318±52	3.443	
^{18}F	975	1100±50	3.086	
^{19}F	985	1043±24	2.829	
^{20}F	950	1113±11	3.025	
^{21}F	1000	1099±12	2.903	
^{23}F	1020	1148±16	2.960	
^{24}F	1005	1253±23	3.238	
^{25}F	1010	1298±31	3.319	
^{26}F	950	1353±54	3.464	

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